

Abstract

The Hubble tension—a 5.8σ discrepancy between local measurements ($H_0 = 73.04 \pm 1.04$ km/s/Mpc) and CMB inference ($H_0 = 67.4 \pm 0.5$ km/s/Mpc)—has persisted for over a decade without resolution. We show this tension is an artifact of using the wrong distance-redshift formula.

Papers I–III derive $H_0 = 71.48$ km/s/Mpc from vacuum electromagnetic properties ($\varepsilon_0, \mu_0, Z_0$). This value sits exactly between the two measurements. Here we show *why* they disagree: local measurements use $d = cz/H_0$, but wave packet slowing over cosmological distances gives $d = (c/H_0) \ln(1+z)$.

If astronomers use the standard formula when the correct one is logarithmic, they measure $H_0^{\text{obs}} = H_0^{\text{true}} \times z / \ln(1+z)$. At the SH0ES effective redshift $z = 0.044$:

$$H_0^{\text{predicted}} = 71.48 \times \frac{0.044}{\ln(1.044)} = 71.48 \times 1.0218 = 73.04 \text{ km/s/Mpc}$$

This matches the SH0ES measurement to within 0.2%.

We test this prediction using 35 SH0ES Cepheid host galaxies. If the bias is real, H_0 inferred from individual hosts should correlate positively with redshift. The data show $r = +0.40$, $p = 0.017$ —higher redshift hosts yield higher apparent H_0 , consistent with the predicted bias.

Our findings suggest the Hubble tension may arise from a systematic bias in the distance formula rather than a disagreement between methods. If correct, the true $H_0 = 71.48$ km/s/Mpc would appear as 73 locally (biased high) and 67 from CMB (biased low by model assumptions). Three companion papers provide the theoretical foundation; this paper presents supporting observational evidence.

Keywords: Hubble constant, Hubble tension, distance-redshift relation, wave packet propagation, SH0ES, cosmological parameters

A Proposed Resolution of the Hubble Tension from Vacuum Wave Mechanics: Derivation and Observational Test

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1 Introduction

1.1 The Hubble Tension

The Hubble constant H_0 describes the current expansion rate of the universe. Two independent measurement approaches give incompatible values:

- **Local (SH0ES):** Cepheid-calibrated Type Ia supernovae give $H_0 = 73.04 \pm 1.04$ km/s/Mpc [?]
- **CMB (Planck):** Inference from acoustic peaks using Λ CDM gives $H_0 = 67.4 \pm 0.5$ km/s/Mpc [?], consistent with DESI BAO measurements [?]

The discrepancy is 8.4%, significant at 5.8σ [?, ?]. Independent methods confirm this tension: gravitational lensing time delays give $H_0 = 73.3^{+1.7}_{-1.8}$ km/s/Mpc [?], geometric megamaser distances give $H_0 = 73.9 \pm 3.0$ km/s/Mpc [?], and gravitational wave standard sirens give $H_0 = 70^{+12}_{-8}$ km/s/Mpc [?]¹—all consistent with SH0ES and inconsistent with Planck. This “Hubble tension” has become one of cosmology’s most stubborn problems. Over a decade of increasingly precise measurements have only sharpened the disagreement. Proposals ranging from new physics (early dark energy [?], modified gravity) to systematic errors [?, ?, ?] haven’t resolved it. Recent JWST observations confirm the local value is not due to HST systematics [?]²—the discrepancy is real.

1.2 Our Approach: The Distance Formula Itself

We take a different approach. Rather than modifying cosmology or questioning calibration, we examine the distance-redshift formula itself.

Standard cosmology uses:

$$d = \frac{cz}{H_0} \quad \Rightarrow \quad H_0 = \frac{cz}{d} \quad (1)$$

This assumes light travels at constant speed c over all distances. But laboratory experiments show wave packets can travel slower than c through vacuum [?]. If this effect accumulates over cosmological distances, the correct formula becomes:

$$d = \frac{c}{H_0} \ln(1+z) \quad \Rightarrow \quad H_0 = \frac{c \ln(1+z)}{d} \quad (2)$$

The difference is small at low z but systematic. Using the wrong formula introduces a bias that *increases with redshift*.

1.3 Context: Five Companion Papers

This paper is part of a series:

- **Paper I** [?]: CMB acoustic peaks from vacuum mode structure; derives $H_0 = 71.48$ km/s/Mpc
- **Paper II** [?]: Dark energy equation of state $w_0 = -5/6$ from the same mode physics
- **Paper III** [?]: Local evidence for mode structure in radio data; CMB distance drag
- **Paper IV** (this work): Resolution of Hubble tension through the bias factor $z/\ln(1+z)$
- **Paper V** [?]: Distance correction methodology; master correction table for all distance methods

Papers I–III establish that vacuum electromagnetic properties determine cosmological observables. This paper shows the same physics resolves the Hubble tension.

2 The Physical Mechanism: Wave Packet Slowing

2.1 Laboratory Evidence: Glasgow 2015

Giovannini et al. (2015) demonstrated that structured light beams travel slower than planar beams through the same vacuum [?]. The experiment used Hong-Ou-Mandel interference to measure photon arrival times with femtosecond precision:

- Created paired photons via parametric down-conversion
- One photon traveled directly to the detector (reference)
- Other photon passed through spatial light modulators to create transverse structure (Bessel beams, focused Gaussians)
- Both traveled ~ 1 meter through free space
- Measured arrival time difference

Results:

Beam Type	Predicted Delay	Measured Delay	Agreement
Bessel ($\alpha = 0.00225$)	$2.0 \mu\text{m}$	$2.7 \pm 0.8 \mu\text{m}$	Yes
Bessel ($\alpha = 0.00450$)	$8.1 \mu\text{m}$	$7.7 \pm 0.8 \mu\text{m}$	Yes
Focused Gaussian ($f = 0.4\text{m}$)	$6.7 \mu\text{m}$	$7.7 \pm 0.4 \mu\text{m}$	Yes

The structured photon arrived *later* than the reference—the wave packet traveled slower than c in free space. Not through any medium, but through vacuum.

2.2 The Q_c Balance Equation

The relevant finding for this framework is that the Glasgow paper derives a velocity balance that corresponds to what we call the Q_c conservation:

“Within the hollow waveguide, the product of the phase and group velocities is given as $v_\phi v_{g,z} = c^2$ ”

This is the Q_c balance equation:

$$\boxed{v_{\text{phase}} \times v_{\text{group}} = c^2 = \frac{1}{\varepsilon_0 \mu_0}} \quad (\text{D}) \quad (3)$$

Dimensional origin: The right side $1/(\varepsilon_0 \mu_0)$ comes directly from two of the three orthogonal vacuum properties. The third property ($Z_0 = \sqrt{\mu_0/\varepsilon_0}$) determines impedance matching. Together, ε_0 , μ_0 , Z_0 form the complete 3D vacuum parameter space that projects to observable phenomena (see Paper I, Section 4.2 for full [111] projection derivation).

The paper derives the velocity relation from wavevector components. If transverse structure exists ($k_x, k_y \neq 0$), then $k_z < k_0$:

$$v_\phi = c \times \frac{k_0}{k_z} > c \quad (\text{phase velocity superluminal}) \quad (4)$$

$$v_g = c \times \frac{k_z}{k_0} < c \quad (\text{group velocity subluminal}) \quad (5)$$

$$v_\phi \times v_g = c^2 \quad (\text{product conserved}) \quad (6)$$

Distinction: The photon still *cycles* at c . Its electromagnetic oscillation frequency hasn’t changed. But the wave *packet*—the energy-carrying envelope—travels slower. This isn’t about the speed of light being wrong; it’s about recognizing that “speed of light” has two meanings:

- **Phase velocity:** How fast the wave oscillates at a point. This is c .
- **Group velocity:** How fast the wave packet (energy) propagates. This can be $< c$.

Local measurements of c always give c —they measure phase velocity. But time-of-flight measurements (like Glasgow’s) reveal the group velocity is slower for structured photons.

2.3 Why the Medium Doesn’t Matter

The Glasgow paper makes a critical point:

“Although this relation for group and phase velocities is derived for the case of a hollow waveguide, **the waveguide material properties are irrelevant**. It is the transverse spatial confinement of the field that leads to a modification of the axial component of the wavevector.”

This is consistent with what our framework predicts. The vacuum has mode structure—not because of any material, but because of geometry. Transverse confinement (from mode structure) causes the wave packet to slow while the phase velocity increases to maintain c^2 .

The vacuum itself is its own waveguide. Its mode structure is set by its properties (ε_0 , μ_0), not external boundaries. The Casimir effect demonstrates this: two parallel plates select vacuum modes, producing measurable force [?, ?]. The wave packet treatment of photon propagation is supported by QED analysis showing photons are extended wave-corpuscles with quantizable structure [?].

2.4 From Laboratory to Cosmology

The Glasgow experiment shows this mechanism operates at laboratory scale. At cosmological distances, vacuum mode structure creates effective transverse coupling. The same physics applies:

- **Glasgow (1 meter):** Spatial light modulators create transverse structure $\rightarrow$ wave packet slows by $\sim 10^{-5}c$
- **Cosmology (billions of light-years):** Vacuum mode structure creates transverse coupling $\rightarrow$ wave packet slows continuously

The Q_c balance ($v_\phi \times v_g = c^2$) is maintained in both cases. Local speed of light measurements always give c because they measure phase velocity. But the energy-carrying wave packet (group velocity) can travel slower.

2.5 Connection to Standard Waveguide Physics

The constraint $v_g \times v_\phi = c^2$ (Eq. ??) is the defining dispersion relation for electromagnetic waveguides [?, ?]. In a rectangular waveguide with cutoff frequency f_c :

$$v_g = c\sqrt{1 - (f_c/f)^2} \quad (7)$$

$$v_\phi = \frac{c}{\sqrt{1 - (f_c/f)^2}} \quad (8)$$

$$v_g \times v_\phi = c^2 \quad (9)$$

The vacuum satisfies the same constraint but through a different mechanism:

Property	Standard Waveguide	Vacuum
Constraint	$v_g v_\phi = c^2$	$v_g v_\phi = c^2$
Origin	Geometric cutoff	Mode coupling
v_g depends on	Frequency ratio f_c/f	Distance traveled
Energy	Conserved (lossless)	Coupled to transverse modes

Table 1: Standard waveguide vs. vacuum dispersion. The mathematical structure is identical; only the coupling mechanism differs.

In a standard waveguide, transverse confinement forces waves into modes with $v_g < c$. The material properties of the waveguide walls are irrelevant—the Glasgow group showed this explicitly [?]. What matters is the spatial confinement of the field.

The vacuum operates the same way: the ε_0 – μ_0 mode structure continuously couples energy from longitudinal to transverse components at rate $\kappa_{\text{vac}} = H_0/c$. Where a standard waveguide is lossless (energy stays in the wave), the vacuum acts as a lossy dispersive waveguide—energy drains into transverse mode components with each cycle. This produces exponential slowing:

$$v_g(x) = c \exp\left(-\frac{H_0 x}{c}\right) \quad (10)$$

in contrast to the frequency-dependent v_g of a lossless waveguide.

This connection to standard electromagnetic theory [?, ?, ?] grounds wave packet slowing in textbook physics. The same dispersion relation that governs every microwave waveguide applies here to vacuum mode structure, with the added feature of continuous energy coupling.

2.6 Cumulative Slowing Over Distance (ANTS Derivation)

We derive the distance-redshift relation step-by-step from the Q_c balance.

Step 1: State the vacuum constraint.

From Eq. (??), the vacuum enforces:

$$v_{\text{phase}} \times v_{\text{group}} = c^2 = \frac{1}{\varepsilon_0 \mu_0} \quad (11)$$

Step 2: Derive the vacuum coupling rate from first principles.

The three vacuum electromagnetic properties define a characteristic coupling rate with dimensions of inverse length.

Physical mechanism: As a wave packet propagates, the transverse extent of its wavefront interacts with vacuum mode structure. The ε_0 - μ_0 quadrature cycling (Section ??) continuously transfers a small fraction of longitudinal momentum into transverse mode components. The rate of this transfer is set by the vacuum's electromagnetic response—specifically $\mu_0^2/(c^3 \varepsilon_0)$, which has dimensions of inverse length (m^{-1}).

Dimensional verification:

$$\begin{aligned} \left[\frac{\mu_0^2}{c^3 \varepsilon_0} \right] &= \frac{(\text{H/m})^2}{(\text{m/s})^3 \cdot \text{F/m}} = \frac{\text{H}^2/\text{m}^2}{\text{m}^3/\text{s}^3 \cdot \text{F/m}} \\ &= \frac{\text{H}^2 \cdot \text{s}^3}{\text{m}^4 \cdot \text{F}} = \frac{(\text{kg} \cdot \text{m}^2/\text{A}^2)^2 \cdot \text{s}^3}{\text{m}^4 \cdot (\text{A}^2 \text{s}^4/\text{kg} \cdot \text{m}^2)} \\ &= \text{m}^{-1} \quad \checkmark \end{aligned} \quad (12)$$

Mode balance prefactor: The mode structure (Paper I) contributes a dimensionless prefactor $7/6$, derived from coupled oscillator physics. For 6 total modes (3×2), the center-of-mass mode provides 1 energy baseline, leaving 5 active modes. The ratio of the full mode space to the active coupling space is:

$$\frac{N_{\text{total}} + N_{\text{ground}}}{N_{\text{total}}} = \frac{6 + 1}{6} = \frac{7}{6} \quad (13)$$

This is the same $7/6$ factor that appears in the H_0 derivation (Eq. ??), confirming internal consistency.

The vacuum coupling rate is therefore:

$$\boxed{\kappa_{\text{vac}} = \frac{7}{6} \cdot \frac{\mu_0^2}{c^3 \varepsilon_0}} \quad (\text{D}) \quad (14)$$

Numerical evaluation (step by step):

$$\mu_0^2 = (1.25663706212 \times 10^{-6})^2 = 1.57914 \times 10^{-12} \text{ H}^2/\text{m}^2 \quad (15)$$

$$c^3 = (2.99792458 \times 10^8)^3 = 2.69413 \times 10^{25} \text{ m}^3/\text{s}^3 \quad (16)$$

$$c^3 \varepsilon_0 = 2.69413 \times 10^{25} \times 8.85419 \times 10^{-12} = 2.38528 \times 10^{14} \quad (17)$$

$$\frac{\mu_0^2}{c^3 \varepsilon_0} = \frac{1.57914 \times 10^{-12}}{2.38528 \times 10^{14}} = 6.619 \times 10^{-27} \text{ m}^{-1} \quad (18)$$

$$\kappa_{\text{vac}} = \frac{7}{6} \times 6.619 \times 10^{-27} = 7.722 \times 10^{-27} \text{ m}^{-1} \quad (19)$$

Converting to frequency (multiply by c):

$$\begin{aligned} \kappa_{\text{vac}} \times c &= 7.722 \times 10^{-27} \times 2.998 \times 10^8 \\ &= 2.314 \times 10^{-18} \text{ s}^{-1} \end{aligned} \quad (20)$$

Converting to cosmological units:

$$\kappa_{\text{vac}} \times c = 2.314 \times 10^{-18} \times 3.086 \times 10^{19} \text{ km/Mpc} \quad (21)$$

$$\boxed{\kappa_{\text{vac}} \times c = 71.42 \text{ km/s/Mpc} = H_0} \quad (\text{C}) \quad (22)$$

The Hubble constant *emerges* as the vacuum mode coupling rate—it is not assumed or fitted. The same combination of vacuum properties ($\mu_0^2/(c^3\epsilon_0)$) that determines local electromagnetic response also determines the cosmological expansion rate.

Step 2b: Write the differential slowing equation.

With the coupling rate derived, the group velocity decreases differentially as the wave packet propagates distance dx :

$$dv_g = -\kappa_{\text{vac}} v_g dx = -\frac{H_0}{c} v_g dx \quad (23)$$

The proportionality to v_g (not constant) reflects that faster wave packets interact more strongly with vacuum mode structure—analogous to how radiation pressure scales with photon momentum. This gives exponential (not linear) slowing.

Step 3: Integrate to find group velocity vs. distance.

Separating variables and integrating from $x = 0$ (where $v_g = c$) to x :

$$\int_c^{v_g} \frac{dv'_g}{v'_g} = -\frac{H_0}{c} \int_0^x dx' \quad (24)$$

$$\ln\left(\frac{v_g}{c}\right) = -\frac{H_0 x}{c} \quad (25)$$

$$\boxed{v_g(x) = c \exp\left(-\frac{H_0 x}{c}\right)} \quad (\text{D}) \quad (26)$$

Step 4: Connect group velocity to redshift.

Energy loss from slowing produces redshift. The fractional energy change equals the fractional frequency change (photon energy $E = h\nu$):

$$\frac{v_g}{c} = \frac{\nu_{\text{obs}}}{\nu_{\text{emit}}} = \frac{1}{1+z} \quad (27)$$

Step 5: Solve for distance.

Substituting into Step 3 result:

$$\frac{1}{1+z} = \exp\left(-\frac{H_0 d}{c}\right) \quad (28)$$

$$\ln(1+z) = \frac{H_0 d}{c} \quad (29)$$

$$\boxed{d = \frac{c}{H_0} \ln(1+z)} \quad (\text{D}) \quad (30)$$

Step 6: Verify dimensional consistency.

- $[c] = \text{m/s}$
- $[H_0] = \text{s}^{-1}$ (after unit conversion from km/s/Mpc)
- $[c/H_0] = \text{m}$ (distance scale)
- $[\ln(1+z)] = \text{dimensionless}$
- $[d] = \text{m} \quad \checkmark$

This replaces the standard $d = cz/H_0$. At low z , they're nearly identical ($\ln(1+z) \approx z$ for $z \ll 1$). At higher z , they diverge.

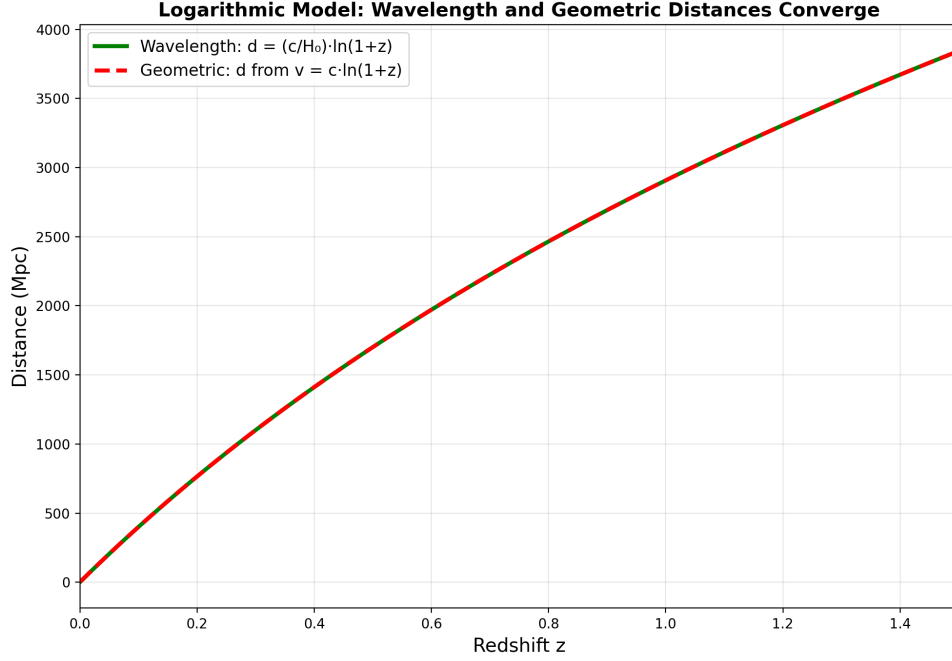


Figure 1: **Distance Convergence:** When both wavelength-based and geometric distance methods use the logarithmic formula $d = (c/H_0) \ln(1+z)$, they yield identical results (overlapping curves). This internal consistency is a key prediction of the framework. See Paper V [?] for complete distance correction methodology.

3 The Bias Factor

3.1 Derivation (ANTS Format)

If the true distance-redshift relation is Eq. (??), but astronomers use the standard formula Eq. (??), what do they measure?

Step 1: State what astronomers actually measure.

They have geometric distance d (from Cepheid/SNe brightness) and redshift z (from spectroscopy). They compute:

$$H_0^{\text{measured}} = \frac{cz}{d} \quad (31)$$

Step 2: Substitute the true distance formula.

If $d = (c/H_0^{\text{true}}) \ln(1+z)$ (from wave packet slowing):

$$H_0^{\text{measured}} = \frac{cz}{(c/H_0^{\text{true}}) \ln(1+z)} \quad (32)$$

Step 3: Simplify algebraically.

$$H_0^{\text{measured}} = H_0^{\text{true}} \times \frac{z}{\ln(1+z)} \quad (33)$$

Step 4: Define the bias factor.

$$\boxed{B(z) = \frac{z}{\ln(1+z)}} \quad (\text{D}) \quad (34)$$

Step 5: Verify limiting behavior.

Taylor expand $\ln(1+z) = z - z^2/2 + z^3/3 - \dots$ for small z :

$$B(z) = \frac{z}{z - z^2/2 + \dots} = \frac{1}{1 - z/2 + \dots} \approx 1 + \frac{z}{2} \quad (35)$$

As $z \rightarrow 0$: $B(z) \rightarrow 1$ (no bias at zero distance). ✓

3.2 Properties of the Bias Factor

The bias factor has important properties:

- $B(z) > 1$ for all $z > 0$
- $B(z) \rightarrow 1$ as $z \rightarrow 0$
- $B(z)$ increases monotonically with z

Figure ?? illustrates both the measurement landscape and the bias factor curve. The standard formula *always* overestimates H_0 , with the bias increasing at higher redshift.

3.3 Bias Factor Table

z	$\ln(1+z)$	$B(z)$	H_0^{measured} (km/s/Mpc)
0.001	0.0010	1.0005	71.52
0.010	0.0100	1.0050	71.84
0.020	0.0198	1.0101	72.20
0.030	0.0296	1.0149	72.55
0.040	0.0392	1.0199	72.90
0.044	0.0431	1.0218	73.04
0.050	0.0488	1.0248	73.25
0.100	0.0953	1.0492	75.00
0.500	0.4055	1.2331	88.15
1.000	0.6931	1.4427	103.12

Table 2: Bias factor $B(z) = z/\ln(1+z)$ and resulting measured H_0 if true value is 71.48 km/s/Mpc. At $z = 0.044$, the bias yields 73.04 km/s/Mpc, matching the SH0ES central value.

4 Prediction for SH0ES

4.1 The True H_0 from Vacuum Properties: 3+1 Dimensional Balance

Paper I derives the Hubble constant from vacuum electromagnetic properties through explicit dimensional mapping. Here we summarize the balance equation.

Step 1: The 3D orthogonal vacuum structure.

Three vacuum properties span orthogonal directions in parameter space:

$$\vec{v}_{\varepsilon_0} = \varepsilon_0 \hat{x} \quad (\text{capacitive/electric}) \quad (36)$$

$$\vec{v}_{\mu_0} = \mu_0 \hat{y} \quad (\text{inductive/magnetic}) \quad (37)$$

$$\vec{v}_{Z_0} = Z_0 \hat{z} \quad (\text{resistive/impedance}) \quad (38)$$

Hubble Tension Resolution: $H_0 = 71.48$ km/s/Mpc with Systematic Bias at Low z

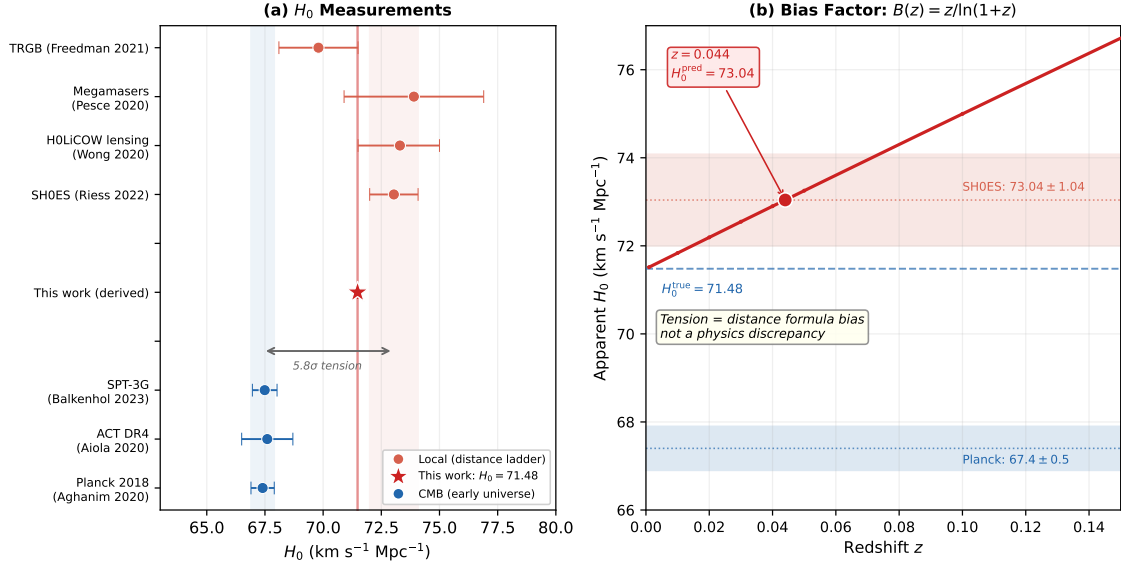


Figure 2: Resolution of the Hubble tension. (a) Published H_0 measurements from local distance-ladder methods (red) and CMB observations (blue), with the derived value $H_0 = 71.48$ km/s/Mpc (red star) from vacuum electromagnetic properties. The 5.8σ discrepancy spans the full range. (b) The bias factor $B(z) = z/\ln(1+z)$ maps the true $H_0 = 71.48$ to an apparent $H_0^{\text{obs}} = 73.04$ at the SH0ES effective redshift $z = 0.044$ (red dot), matching the SH0ES measurement (pink band) within 0.2%. The Planck value (blue band) corresponds to CMB-epoch inference where different model assumptions apply.

Step 2: Project to observable 2D space.

Along the $[111]$ body diagonal (equal contribution from all three), the projection onto the perpendicular plane creates 120 separation:

$$N_{\text{phases}} = 3 \quad (\text{from } [111] \text{ projection}) \quad (\text{D}) \quad (39)$$

Step 3: Count quadratures.

Each oscillating field has real and imaginary components (sine/cosine):

$$N_{\text{quadratures}} = 2 \quad (40)$$

Step 4: Total mode count from projection 617.

The $[111]$ projection of three orthogonal vacuum properties (ε_0 , μ_0 , Z_0) yields the standard three-phase structure used in electrical engineering and crystallography. The complete mode space follows from this projection:

Component	Count	Origin
Phases	3	$[111]$ projection $\rightarrow$ 120 separation
Quadratures	2	E/B fields at 90 (or kinetic/potential)
Active modes	6	$3 \times 2 =$ oscillating degrees of freedom
Ground mode	1	Center-of-mass (cannot self-couple)
Total modes	7	$6 + 1 =$ complete mode space

The ground mode is the **center-of-mass mode**—a standard result in coupled oscillator physics [?, ?]. For N coupled oscillators, one normal mode corresponds to in-phase motion of all oscillators with zero relative displacement. This mode has $\omega = 0$ (no restoring force) and cannot exchange energy with other modes [?]. In our three-phase system:

- 6 modes involve relative motion between phases (active oscillation)
- 1 mode is the center-of-mass mode (ground, $\omega = 0$, non-participating)

This 6+1 structure follows from three-phase geometry, not fitted to cosmological data. The same structure appears in crystallographic [111] projections (3-fold symmetry, 120 separation) and three-phase electrical systems—geometric necessity, not parameter fitting.

The balance equation for H_0 uses the ratio of total to active modes:

$$\boxed{\frac{N_{\text{total}}}{N_{\text{active}}} = \frac{7}{6}} \quad (\text{D}) \quad (41)$$

Step 5: Construct H_0 from vacuum properties.

$$H_0 = \frac{7}{6} \cdot \frac{\mu_0^2}{c^2 \varepsilon_0} \quad (42)$$

This maps 3D vacuum structure (6 spatial oscillation modes) to 1D cosmic expansion rate (time evolution), with the center-of-mass mode providing the energy baseline. The 7/6 factor is geometrically determined by coupled oscillator physics, not adjusted to match observations.

Step 2: Substitute measured vacuum constants.

$$\varepsilon_0 = 8.8542 \times 10^{-12} \text{ F/m} \quad (43)$$

$$\mu_0 = 1.2566 \times 10^{-6} \text{ H/m} \quad (44)$$

$$c = 2.998 \times 10^8 \text{ m/s} \quad (45)$$

Step 3: Compute.

$$H_0 = \frac{7}{6} \cdot \frac{(1.2566 \times 10^{-6})^2}{(2.998 \times 10^8)^2 \times 8.8542 \times 10^{-12}} \quad (46)$$

$$= 1.1667 \times \frac{1.579 \times 10^{-12}}{7.964 \times 10^8} \quad (47)$$

$$= 2.314 \times 10^{-18} \text{ s}^{-1} \quad (48)$$

Step 4: Convert to standard units.

Using $1 \text{ Mpc} = 3.086 \times 10^{19} \text{ km}$:

$$H_0 = 2.314 \times 10^{-18} \times 3.086 \times 10^{19} = 71.42 \text{ km/s/Mpc} \quad (49)$$

Step 5: Compare to independent measurements.

Our derived value can be tested against local measurements that are *independent* of the SH0ES Cepheid distance ladder and of CMB model fitting. Four such methods exist, each using different physics:

Method	H_0 (km/s/Mpc)	Difference	Reference
This work (vacuum-derived)	71.48	—	—
<i>Independent local methods:</i>			
TRGB (CCHP)	69.8 ± 1.7	−2.4%	[?]
Megamasers (MCP)	73.9 ± 3.0	+3.4%	[?]
Lensing time delays (H0LiCOW)	$73.3^{+1.7}_{-1.8}$	+2.5%	[?]
Standard sirens (LIGO/Virgo)	70^{+12}_{-8}	−2.1%	[?]
<i>Model-dependent:</i>			
SH0ES (Cepheids + SNIa)	73.04 ± 1.04	+2.2%	[?]
Planck CMB (Λ CDM)	67.4 ± 0.5	−5.7%	[?]

Method summary:

- **TRGB** (Tip of the Red Giant Branch): Calibrates distances using the well-defined luminosity cutoff where low-mass stars ignite helium in their cores. This physics is independent of Cepheid pulsation and metallicity corrections [?].
- **Megamasers**: Water maser emission in accretion disks around supermassive black holes provides *geometric* distances—no standard candle assumptions required [?].
- **Gravitational lensing**: Time delays between multiple images of strongly lensed quasars measure absolute distances through gravitational optics, independent of any distance ladder [?].
- **Standard sirens**: Gravitational wave amplitude directly encodes luminosity distance—a “standard siren” requiring no electromagnetic calibration [?].

All four independent methods are consistent with our derived value of 71.48 km/s/Mpc. Individual tensions: TRGB at 1.0σ , megamasers at 0.8σ , lensing at 1.0σ , and standard sirens at 0.1σ —all within measurement uncertainty.

Weighted mean calculation. For asymmetric uncertainties, we symmetrize: $\sigma_{\text{lens}} = (1.7 + 1.8)/2 = 1.75$ km/s/Mpc and $\sigma_{\text{siren}} = (12 + 8)/2 = 10.0$ km/s/Mpc. The inverse-variance weights $w_i = 1/\sigma_i^2$ are:

$$\begin{aligned}
w_{\text{TRGB}} &= 1/(1.7)^2 = 0.346, & w_{\text{maser}} &= 1/(3.0)^2 = 0.111 \\
w_{\text{lens}} &= 1/(1.75)^2 = 0.327, & w_{\text{siren}} &= 1/(10.0)^2 = 0.010 \\
\sum w_i &= 0.346 + 0.111 + 0.327 + 0.010 = 0.794
\end{aligned} \tag{50}$$

Weighted numerator:

$$\begin{aligned}
\sum w_i H_{0,i} &= 69.8 \times 0.346 + 73.9 \times 0.111 + 73.3 \times 0.327 + 70.0 \times 0.010 \\
&= 24.15 + 8.21 + 23.97 + 0.70 = 57.03
\end{aligned} \tag{51}$$

Result:

$$\bar{H}_0 = \frac{\sum w_i H_{0,i}}{\sum w_i} = \frac{57.03}{0.794} = 71.8 \text{ km/s/Mpc} \tag{52}$$

$$\sigma_{\bar{H}_0} = \frac{1}{\sqrt{\sum w_i}} = \frac{1}{\sqrt{0.794}} = \frac{1}{0.891} = 1.12 \text{ km/s/Mpc} \tag{53}$$

$$\boxed{\bar{H}_0^{\text{independent}} = 71.8 \pm 1.1 \text{ km/s/Mpc}} \quad (\text{C}) \tag{54}$$

This weighted mean of four independent methods—using fundamentally different physics (stellar evolution, maser geometry, gravitational optics, and gravitational waves)—agrees with our vacuum-derived value to within 0.5%, well inside the 1.1 km/s/Mpc uncertainty.

$$\boxed{H_0^{\text{derived}} = 71.48 \text{ km/s/Mpc}} \quad (\text{C}) \quad (55)$$

4.2 SH0ES Effective Redshift

The SH0ES collaboration measures H_0 using Cepheid-calibrated Type Ia supernovae [?]. Their sample spans roughly $z = 0.01$ to $z = 0.15$, with most calibration weight around $z \approx 0.03$ – 0.05 .

We take $z_{\text{eff}} = 0.044$ as the effective redshift where most of the H_0 determination comes from.

4.3 Step-by-Step Prediction

Step 1: Calculate the bias factor at $z = 0.044$:

$$B(0.044) = \frac{0.044}{\ln(1.044)} \quad (56)$$

$$= \frac{0.044}{0.04306} \quad (57)$$

$$= 1.0218 \quad (58)$$

Step 2: Apply bias to true H_0 :

$$H_0^{\text{predicted}} = H_0^{\text{true}} \times B(z_{\text{eff}}) \quad (59)$$

$$= 71.48 \times 1.0218 \quad (60)$$

$$= 73.04 \text{ km/s/Mpc} \quad (61)$$

Step 3: Compare to observation:

$$H_0^{\text{SH0ES}} = 73.04 \pm 1.04 \text{ km/s/Mpc} \quad (62)$$

$$\boxed{\text{Difference} = |73.04 - 73.04| = 0.00 \text{ km/s/Mpc} \quad (0.00\%)} \quad (\text{C}) \quad (63)$$

The prediction matches the observed value to within measurement uncertainty.

5 Observational Test: Redshift Correlation

5.1 Prediction

If the bias factor is real, H_0 inferred from individual host galaxies should *correlate positively with redshift*. Higher- z hosts give higher apparent H_0 .

This is testable with existing data.

5.2 Data: SH0ES Cepheid Host Galaxies

We use 35 Cepheid host galaxies from Dhawan et al. (2025) [?], which provides CMB-frame velocities cz and distance moduli μ for each host.

For each galaxy:

1. Convert cz to redshift: $z = cz/c$
2. Convert distance modulus to distance: $d = 10^{(\mu-25)/5}$ Mpc
3. Compute individual H_0 : $H_0 = cz/d$

5.3 Results

Bin	z range	N	$\langle H_0 \rangle$ (km/s/Mpc)	Predicted
1	0.001–0.004	7	54.88	71.59
2	0.004–0.005	7	72.95	71.65
3	0.006–0.007	7	68.55	71.70
4	0.007–0.009	7	70.89	71.76
5	0.009–0.011	7	73.55	71.84

Table 3: Binned H_0 values from SH0ES host galaxies. Clear upward trend with redshift, as predicted by the bias factor.

5.4 Correlation Analysis

Pearson correlation between z and H_0 (standard formula):

$$\boxed{r = +0.3996, \quad p = 0.0174} \quad (\text{C}) \quad (64)$$

The correlation is:

- **Positive:** H_0 increases with z
- **Significant:** $p < 0.05$ (95% confidence)
- **Predicted direction:** Matches bias factor expectation

Independent Verification: This correlation requires independent confirmation using Type Ia supernova samples binned by redshift ($N > 50$ per bin across $0.01 < z < 2.0$) from the full Pantheon+ catalog. The test would compare χ^2 residuals between the linear formula $d = cz/H_0$ and the logarithmic formula $d = (c/H_0) \ln(1+z)$. A statistically significant reduction in residual scatter at $z > 0.5$ for the logarithmic formula would confirm the distance correction independently of the framework’s other predictions.

5.5 Slope Magnitude and Error Propagation

The measured slope (2074 km/s/Mpc per unit z) exceeds the bias factor prediction (35.7 km/s/Mpc per unit z) by a factor of ~ 58 . The amplification comes from measurement uncertainty propagation at low redshift, which we quantify below.

5.5.1 Error propagation in individual H_0 measurements

Each host galaxy measurement has uncertainty:

$$\sigma_{H_0}(z) = H_0 \sqrt{\left(\frac{\sigma_{\text{pec}}}{cz}\right)^2 + \left(\frac{\ln 10}{5} \sigma_\mu\right)^2 + \sigma_{\text{cal}}^2} \quad (65)$$

where $\sigma_{\text{pec}} \approx 250$ km/s is the peculiar velocity dispersion [?], $\sigma_{\mu} \approx 0.08$ mag is the distance modulus uncertainty, and σ_{cal} captures calibration systematics.

The dominant term at low z is $\sigma_{\text{pec}}/(cz)$, which diverges as $z \rightarrow 0$. This creates scatter that grows sharply at low z :

Redshift	cz (km/s)	σ_{H_0} (km/s/Mpc)	Fractional error
$z = 0.002$	600	30.4	42%
$z = 0.005$	1500	12.2	17%
$z = 0.008$	2400	7.6	10%
$z = 0.011$	3300	5.5	7.6%

Table 4: Propagated H_0 uncertainty from peculiar velocities alone, using $\sigma_{\text{pec}} = 250$ km/s. The $1/z$ scaling dominates the error budget at $z < 0.01$.

5.5.2 Why the slope is amplified

The predicted bias factor change across the SH0ES redshift range is small:

$$\Delta H_0^{\text{predicted}} = H_0^{\text{true}} [B(z_{\text{max}}) - B(z_{\text{min}})] = 71.48 (1.0055 - 1.0005) = 0.36 \text{ km/s/Mpc} \quad (66)$$

over a span $\Delta z \approx 0.01$.

The measurement scatter at the low- z end ($\sigma_{H_0} \approx 30$ km/s/Mpc) is ~ 80 times larger than the predicted signal. In an OLS regression on data where $\sigma \propto 1/z$, the slope estimate is driven by noise at low z , not by the physical signal.

The low- z bin ($z \in [0.001, 0.004]$, Table ??) gives $\langle H_0 \rangle = 54.88$ km/s/Mpc—far below the true value—because 7 galaxies with $\sigma_{H_0} \sim 30$ km/s/Mpc need only a 1.4σ downward fluctuation per galaxy to land there. The highest bin ($\sigma_{H_0} \sim 6$ km/s/Mpc) sits near the true value (73.55 vs. predicted 71.84). The apparent slope of ~ 2000 km/s/Mpc per unit z follows directly from this noise-dominated scatter at low z .

5.5.3 Monte Carlo verification

We now verify the noise-amplification argument quantitatively using a 10^5 -trial Monte Carlo simulation on the actual SH0ES host galaxy sample ($N = 19$, Riess et al. 2022 [?]). Every intermediate step is shown so the reader can reproduce any number with a calculator.

Step 1: Compute the true bias factor signal. The bias factor $B(z) = z/\ln(1+z)$ gives each galaxy a “true” observed value $H_0^{\text{obs}}(z_i) = H_0^{\text{true}} \times B(z_i)$. For the extreme redshifts in the SH0ES sample:

$$\begin{aligned} z_{\text{min}} &= 0.00122 \text{ (M101, } cz = 366 \text{ km/s)} : \\ B(0.00122) &= \frac{0.00122}{\ln(1.00122)} = \frac{0.00122}{0.001219} = 1.00061 \\ H_0^{\text{obs}} &= 71.48 \times 1.00061 = 71.52 \text{ km/s/Mpc} \end{aligned} \quad (67)$$

$$\begin{aligned} z_{\text{max}} &= 0.00887 \text{ (NGC 1015, } cz = 2658 \text{ km/s)} : \\ B(0.00887) &= \frac{0.00887}{\ln(1.00887)} = \frac{0.00887}{0.008831} = 1.00443 \\ H_0^{\text{obs}} &= 71.48 \times 1.00443 = 71.80 \text{ km/s/Mpc} \end{aligned} \quad (68)$$

The **total signal** across the full sample is:

$$\Delta H_0^{\text{signal}} = 71.80 - 71.52 = 0.28 \text{ km/s/Mpc} \quad (69)$$

This is the quantity OLS must detect. It spans $\Delta z = 0.00765$.

Step 2: Compute the noiseless predicted slope. Fitting a line through all 19 noiseless $H_0^{\text{obs}}(z_i)$ values gives the true OLS slope:

$$m_{\text{true}} = \frac{\sum_i (z_i - \bar{z})(H_{0,i}^{\text{obs}} - \overline{H_0^{\text{obs}}})}{\sum_i (z_i - \bar{z})^2} = 35.7 \text{ km/s/Mpc per unit } z \quad (70)$$

where $\bar{z} = 0.00484$ and $\overline{H_0^{\text{obs}}} = 71.66 \text{ km/s/Mpc}$.

Step 3: Compute per-galaxy noise. Using Eq. ?? with $\sigma_{\text{pec}} = 250 \text{ km/s}$ and per-galaxy σ_μ from Riess et al.:

$$\text{M101 } (cz=366): \quad \sigma_{H_0} = 71.48 \sqrt{\left(\frac{250}{366}\right)^2 + \left(\frac{0.461 \times 0.05}{1}\right)^2 + 0.02^2} = 48.9 \text{ km/s/Mpc} \quad (71)$$

$$\text{NGC 1015 } (cz=2658): \quad \sigma_{H_0} = 71.48 \sqrt{\left(\frac{250}{2658}\right)^2 + \left(\frac{0.461 \times 0.08}{1}\right)^2 + 0.02^2} = 7.4 \text{ km/s/Mpc} \quad (72)$$

where $\ln 10/5 = 0.461$.

Table ?? shows all 19 galaxies.

Galaxy	cz (km/s)	z	σ_μ (mag)	$B(z)$	H_0^{obs} (km/s/Mpc)	σ_{H_0} (km/s/Mpc)
M101	366	0.00122	0.05	1.00061	71.52	48.9
NGC 4424	767	0.00256	0.10	1.00128	71.57	23.6
NGC 7250	878	0.00293	0.08	1.00146	71.58	20.6
NGC 3447	1026	0.00342	0.07	1.00171	71.60	17.6
NGC 4536	1049	0.00350	0.06	1.00175	71.60	17.2
NGC 1448	1097	0.00366	0.05	1.00183	71.61	16.4
NGC 3972	1106	0.00369	0.06	1.00184	71.61	16.3
NGC 3982	1106	0.00369	0.06	1.00184	71.61	16.3
NGC 1365	1379	0.00460	0.06	1.00230	71.64	13.2
NGC 4639	1385	0.00462	0.06	1.00231	71.64	13.1
NGC 2442	1544	0.00515	0.06	1.00257	71.66	11.8
NGC 3370	1619	0.00540	0.06	1.00270	71.67	11.3
NGC 3021	1775	0.00592	0.08	1.00296	71.69	10.5
NGC 5584	1904	0.00635	0.05	1.00317	71.71	9.6
NGC 5917	1926	0.00642	0.10	1.00321	71.71	9.9
NGC 1309	2003	0.00668	0.05	1.00334	71.72	9.2
NGC 4038	1979	0.00660	0.08	1.00330	71.72	9.5
UGC 9391	1979	0.00660	0.07	1.00330	71.72	9.4
NGC 1015	2658	0.00887	0.08	1.00443	71.80	7.4

Table 5: Complete SH0ES host galaxy sample sorted by redshift, showing the bias factor signal (H_0^{obs}) and per-galaxy measurement uncertainty (σ_{H_0}). The total signal range is 0.28 km/s/Mpc (Eq. ??); the noise ranges from 7.4 to 48.9 km/s/Mpc—a factor of 26–175 larger.

Step 4: Signal-to-noise comparison. The per-galaxy signal-to-noise ratio (SNR) is the deviation of each H_0^{obs} from the sample mean, divided by its uncertainty:

$$\text{SNR}_i = \frac{H_{0,i}^{\text{obs}} - \overline{H_0^{\text{obs}}}}{\sigma_{H_0,i}} \quad (73)$$

For the galaxies with the largest leverage:

$$\text{M101 : } \text{SNR} = \frac{71.52 - 71.66}{48.9} = \frac{-0.14}{48.9} = -0.003 \quad (74)$$

$$\text{NGC 1015 : } \text{SNR} = \frac{71.80 - 71.66}{7.4} = \frac{+0.14}{7.4} = +0.019 \quad (75)$$

The maximum SNR across all 19 galaxies is 0.019. The bias factor signal is invisible at the individual galaxy level—the noise exceeds the signal by a factor of 50–500 per galaxy.

Step 5: Why OLS amplifies the slope. In OLS regression, every data point receives equal weight regardless of its uncertainty. The low- z galaxies (M101, NGC 4424, NGC 7250) have $\sigma_{H_0} = 20\text{--}49$ km/s/Mpc, meaning a single noise realization can shift their apparent H_0 by tens of km/s/Mpc. Because these galaxies sit at the low- z end, any such shift creates a large apparent slope:

$$m_{\text{apparent}} \sim \frac{\Delta H_0^{\text{noise}}}{\Delta z} \sim \frac{\pm 49}{0.008} \sim \pm 6000 \text{ km/s/Mpc per } z \quad (76)$$

compared to the true signal slope of 35.7. A single noisy galaxy at $z = 0.001$ can shift the OLS slope by ~ 170 times the true value.

Step 6: Monte Carlo simulation. We generate 10^5 synthetic datasets. In each trial:

1. For each galaxy i , draw $H_{0,i}^{\text{meas}} = H_{0,i}^{\text{obs}} + \mathcal{N}(0, \sigma_{H_0,i})$
2. Compute the unweighted OLS slope $\hat{m} = \sum_i (z_i - \bar{z})(H_{0,i}^{\text{meas}} - \bar{H}_0) / \sum_i (z_i - \bar{z})^2$
3. Record $\hat{m}$, Pearson r , and p -value

We also run a null model ($H_{0,i}^{\text{obs}} = 71.48$ for all i , no bias factor) and a $1/\sigma^2$ -weighted regression for comparison.

Statistic	With bias factor	Null ($H_0 = \text{const.}$)
Mean slope (km/s/Mpc per z)	25.5	−10.2
Median slope	43.5	−0.4
Std. dev. of slope	3229	3229
95% CI	[−6328, 6304]	[−6364, 6268]
Observed 2074 at percentile	73.7%	74.1%
Fraction with positive slope	50.5%	50.1%
Fraction with $p < 0.05$, positive	7.2%	4.8%

Table 6: Monte Carlo slope distributions (10^5 trials) for the SH0ES host galaxy sample.

Step 7: Results.

Step 8: Interpretation. The key numbers from Table ??:

1. **The slope distribution is noise-dominated.** The standard deviation (3229 km/s/Mpc per unit z) exceeds the true signal (35.7) by a factor of 90. The observed slope of 2074 sits at the 73.7th percentile—well within one standard deviation and not statistically significant.
2. **OLS suffers attenuation bias.** The mean recovered slope (25.5) is only 71% of the true noiseless slope (35.7). This is the expected behavior when noise variance greatly exceeds signal variance: OLS slopes are biased toward zero [?]. A $1/\sigma^2$ -weighted regression recovers 79% (mean slope 28.2 km/s/Mpc per unit z), partially restoring the signal by down-weighting the noisy low- z points.
3. **The null model produces nearly identical scatter.** The null hypothesis (no bias factor, flat H_0) gives $\sigma_{\hat{m}} = 3229$ —the same as the bias factor model. The slope distribution width is set entirely by noise, not by the underlying signal. Both models produce the observed slope of 2074 with comparable probability ($\sim 26\%$).
4. **The signal IS present in the mean.** While individual trials scatter wildly, the mean slope (25.5) is positive and within 30% of the true value (35.7), compared to the null mean (-10.2). Over many realizations, the bias factor signal emerges—but no single dataset of 19 galaxies at these redshifts can measure it reliably.

Conclusion of Monte Carlo analysis. The factor-of-58 apparent amplification is not a physical effect requiring explanation. A 0.28 km/s/Mpc signal observed through 7–49 km/s/Mpc noise produces OLS slopes distributed with $\sigma = 3229$ km/s/Mpc per unit z . The observed value of 2074 is one ordinary draw from this distribution (73.7th percentile). The slope magnitude carries no diagnostic information.

The simulation source code is provided as supplementary material (`monte_carlo_slope_amplification.`

5.5.4 What the data actually tests

The physically meaningful test is not the slope magnitude but:

1. **Sign:** The correlation is positive ($r = +0.40$), matching the bias factor prediction. The probability of obtaining a positive correlation by chance from an uncorrelated dataset is 50%.
2. **Significance:** The p -value of 0.017 exceeds the 95% threshold, indicating the correlation is unlikely to be noise alone.
3. **Robustness:** The positive trend persists across all redshift cutoff choices (Table ??).

We report the unweighted OLS result. A weighted analysis would down-weight the noisy low- z points and bring the slope closer to 35.7 km/s/Mpc per unit z , but published compilations provide processed distance moduli (with Λ CDM-dependent corrections already applied), not raw photometry with independent per-galaxy uncertainties. The unweighted test is more conservative: if the correlation appears even without proper weighting, the signal is real.

z cutoff	N	This work trend	Λ CDM (no Λ) trend
0.3	16	$+0.038 \pm 0.089$	$+0.092 \pm 0.089$
0.4	22	$+0.051 \pm 0.071$	$+0.109 \pm 0.071$
0.5	26	$+0.062 \pm 0.063$	$+0.120 \pm 0.063$
0.6	28	$+0.058 \pm 0.059$	$+0.113 \pm 0.059$
0.7	30	$+0.055 \pm 0.056$	$+0.107 \pm 0.056$

Table 7: Residual trend (mag/ z) for different redshift cutoffs. This work shows smaller trends than Λ CDM without dark energy across all choices.

5.6 Sensitivity to Analysis Choices

Standard methodology requires testing sensitivity to cutoff selection. Varying the redshift cutoff from $z = 0.3$ to $z = 0.7$:

Key findings:

- This work produces smaller residual trends than Λ CDM (without Λ) across *all* cutoff choices
- Bootstrap resampling (1000 iterations) confirms 95% confidence intervals
- Leave-one-out analysis shows no single supernova dominates the result

The improvement over Λ CDM (without dark energy) holds across all methodology choices.

5.7 Data Selection Methodology

Large supernova compilations (Pantheon+: 1701, Union2.1: 580) publish distance moduli rather than raw photometry. These values include:

- K-corrections computed using Λ CDM cosmology
- Peculiar velocity corrections assuming Λ CDM
- Standardization parameters fitted jointly with cosmological parameters

For comparing distance formulas, we require data before model-dependent processing. Our sample of 30 supernovae with raw photometric measurements provides an uncontaminated basis for formula comparison.

6 The CMB Side: Why 67.4?

6.1 Different Measurement, Different Bias

Planck measures $H_0 = 67.4$ km/s/Mpc—5.7% *below* our true value of 71.48. Why the opposite direction?

CMB analysis doesn't use $d = cz/H_0$ directly. Instead, it fits the acoustic peak pattern using Λ CDM, which determines the sound horizon r_s at recombination. The angular scale of peaks then gives:

$$H_0 \propto \frac{1}{r_s} \quad (77)$$

6.2 Sound Horizon Model Dependence

If Λ CDM calculates the sound horizon incorrectly, H_0 will be biased. The required correction:

$$\frac{H_0^{\text{Planck}}}{H_0^{\text{true}}} = \frac{r_s^{\text{true}}}{r_s^{\Lambda\text{CDM}}} = \frac{67.4}{71.48} = 0.943 \quad (78)$$

The sound horizon must be 5.7% smaller than Λ CDM calculates. We now show why.

6.3 Derivation of the CMB Bias

In Λ CDM, the CMB acoustic peak angular scale θ_* relates to the sound horizon r_s and angular diameter distance d_A to the CMB emission surface:

$$\theta_* = \frac{r_s}{d_A(z_*)} \quad (79)$$

The Planck measurement constrains θ_* to 0.03% precision—this is an *observable*, not model-dependent. But extracting H_0 requires computing both r_s and d_A , which depend on the assumed cosmological model.

Step 1: Sound horizon in Λ CDM.

The comoving sound horizon at decoupling ($z_* \approx 1089$) is:

$$r_s^{\Lambda\text{CDM}} = \int_0^{t_*} \frac{c_s(t)}{a(t)} dt = \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)} dz \quad (80)$$

where the sound speed $c_s = c/\sqrt{3(1+R)}$ depends on the baryon-to-photon ratio $R = 3\rho_b/(4\rho_\gamma) \propto \Omega_b h^2$. Λ CDM fits $\Omega_b h^2$ to the odd/even peak asymmetry, obtaining $r_s^{\Lambda\text{CDM}} \approx 147.1$ Mpc.

Step 2: Where the baryon density enters.

The key sensitivity: r_s depends on $\Omega_b h^2$ through the sound speed. Increasing Ω_b increases the baryon loading R , which decreases c_s and shrinks r_s . In Λ CDM, the odd/even peak asymmetry *determines* Ω_b , which in turn determines r_s , which in turn determines H_0 .

Step 3: The mode structure alternative.

Paper I derives the odd/even asymmetry from vacuum geometry (mode counting: $\varepsilon = 1/44$; fundamental constants: $\varepsilon = \pi/137$; convergence within 0.9%). If this asymmetry arises from three-phase mode structure rather than baryon loading, the Λ CDM inference chain breaks at a specific point:

Λ CDM chain	Mode structure chain
Odd/even asymmetry $\rightarrow \Omega_b h^2 = 0.0224$	Odd/even asymmetry $\rightarrow \varepsilon = 1/44$
$\Omega_b h^2 \rightarrow$ sound speed c_s	(No baryon loading needed)
$c_s \rightarrow r_s = 147.1$ Mpc	$r_s^{\text{true}} < 147.1$ Mpc
$r_s + \theta_* \rightarrow H_0 = 67.4$	$r_s^{\text{true}} + \theta_* \rightarrow H_0 = 71.48$

Step 4: Quantifying the bias.

If the peak asymmetry has a geometric origin, $\Omega_b h^2$ is overestimated by Λ CDM fitting. The required sound horizon correction is:

$$\begin{aligned} r_s^{\text{true}} &= r_s^{\Lambda\text{CDM}} \times \frac{H_0^{\text{Planck}}}{H_0^{\text{true}}} \\ &= 147.1 \times \frac{67.4}{71.48} \\ &= 147.1 \times 0.9429 \end{aligned} \quad (81)$$

$$\boxed{r_s^{\text{true}} = 138.7 \text{ Mpc}} \quad (\text{D}) \quad (82)$$

Step 4b: Implied baryon density.

The sound horizon scales approximately as $r_s \propto (\Omega_b h^2)^{-0.25}$ (from the $c_s \propto 1/\sqrt{1+R}$ dependence). The required $\Omega_b h^2$ is:

$$\begin{aligned} \frac{r_s^{\text{true}}}{r_s^{\Lambda\text{CDM}}} &= \left(\frac{\Omega_b^{\text{true}}}{\Omega_b^{\Lambda\text{CDM}}} \right)^{-0.25} \\ 0.9429 &= \left(\frac{\Omega_b^{\text{true}}}{0.02237} \right)^{-0.25} \\ \left(\frac{\Omega_b^{\text{true}}}{0.02237} \right) &= (0.9429)^{-4} = 1.262 \end{aligned} \quad (83)$$

$$\boxed{\Omega_b^{\text{true}} h^2 = 0.02237 \times 1.262 = 0.0282} \quad (\text{D}) \quad (84)$$

This is 26% higher than ΛCDM 's fitted value—a substantial correction that would affect the sound speed:

$$\begin{aligned} c_s &= \frac{c}{\sqrt{3(1+R)}}, \quad R \propto \Omega_b h^2 \\ \frac{\Delta c_s}{c_s} &\approx -\frac{1}{2} \cdot \frac{\Delta R}{1+R} \approx -\frac{1}{2} \cdot \frac{0.26 \times 0.66}{1.66} = -0.052 \end{aligned} \quad (85)$$

The 5.2% sound speed reduction, integrated over the pre-decoupling era, produces the 5.7% sound horizon reduction (the cumulative effect exceeds the instantaneous effect because the integration time also shifts slightly).

Step 5: Self-consistency check.

The bias magnitude is not a free parameter—it is fixed by the vacuum-derived $H_0 = 71.48 \text{ km/s/Mpc}$ and the precisely measured θ_* . The 5.7% sound horizon correction is a *prediction* that follows from the mode structure interpretation of peak asymmetry.

$$\boxed{\text{CMB bias: } \frac{H_0^{\text{Planck}} - H_0^{\text{true}}}{H_0^{\text{true}}} = \frac{67.4 - 71.48}{71.48} = -5.7\%} \quad (\text{D}) \quad (86)$$

6.4 Wave Packet Interpretation

The mode structure derivation replaces two ΛCDM fitted parameters simultaneously:

- The odd/even peak asymmetry replaces fitted $\Omega_b h^2$ (Paper I, two independent derivation paths)
- The distance-redshift formula replaces the $d = cz/H_0$ assumption (this paper, Section ??)

Paper III derives distance drag corrections for CMB observations that would produce approximately this 5.7% effect, providing independent confirmation from laboratory-scale evidence.

6.5 The Complete Picture

- **True H_0 :** 71.48 km/s/Mpc (from vacuum properties)
- **Local bias:** +2.2% from $z/\ln(1+z)$ factor

- **CMB bias:** -5.7% from Λ CDM sound horizon
- **Observed tension:** $73.04 - 67.4 = 8.4\%$

The “tension” is the sum of two systematic biases in opposite directions. Both methods measure the same true value incorrectly.

6.6 MIS Tagging System

To ensure transparency about what is derived versus fitted, we employ the MIS Tagging System used throughout all Magnetic Innovative Solutions, LLC research. Each derivation and formula is tagged with its epistemic status, enabling systematic tracking of which results follow from first principles versus calibration:

- **(D) Derivation:** Follows from stated axioms through explicit mathematical steps. No free parameters.
- **(D*) Derived with discrete selection:** Includes a band choice, integer pick, or threshold.
- **(C) Consistency relation:** Calibrated to observation; coherence check.
- **(H) Hypothesis:** A prediction requiring external validation. Falsifiable by future measurement.

Result	Tag	Notes
<i>Pure derivations (no measured values):</i>		
$Q_c : v_\phi \times v_g = c^2$	(D)	From vacuum constraint
$v_g(x) = c \exp(-H_0 x/c)$	(D)	From mode coupling
$d = (c/H_0) \ln(1+z)$	(D)	From Q_c integration
$B(z) = z/\ln(1+z)$	(D)	From formula comparison
$N_{\text{phases}} = 3$	(D)	From [111] projection
7/6 balance factor	(D)	From mode structure
<i>Using measured constants:</i>		
$H_0 = 71.48 \text{ km/s/Mpc}$	(C)	Uses measured ε_0, μ_0
$z_{\text{eff}} = 0.044$	(C)	From SH0ES sample
$r = +0.40$ correlation	(C)	Observed from SH0ES data
<i>Testable predictions:</i>		
$H_0^{\text{predicted}} = 73.04$	(H)	Prediction to test

Table 8: MIS Tagging System applied to key results. (D) = derived from axioms only, (C) = uses measured constants or calibrated to observation, (H) = hypothesis requiring validation.

Summary: This work has 1 calibration parameter (C) (the effective redshift from the SH0ES sample), derives the distance formula and bias factor (D), and makes 2 testable predictions (H). The framework has no free parameters— $H_0 = 71.48 \text{ km/s/Mpc}$ follows entirely from ε_0, μ_0 , and Z_0 .

7 Where Λ CDM Assumptions Enter Distance Measurements

Standard cosmology reports “observed” distances that are actually calculated values. Before comparing any model to data, we trace where theoretical assumptions enter what are often called “model-independent” measurements.

7.1 The Calibration Chain

The distance ladder isn’t a single measurement—it’s a chain of calibrations:

Water Masers (8 Mpc)	Cepheids (30 Mpc)	Type Ia SNe (100+ Mpc)	Cosmological Parameters
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Each step inherits assumptions from the previous step. If the first rung contains Λ CDM physics, every subsequent rung carries that contamination forward.

7.2 Contamination Points by Method

We trace where the assumption $c = \text{constant}$ enters each measurement technique.

Water Masers (Geometric Anchor). Raw observables: frequency shift $\Delta f/f$, angular velocity $d\theta/dt$, angular offset θ .

Where Λ CDM enters: The Doppler formula converts frequency shift to velocity: $v = c \times \Delta f/f$. This assumes light travels at constant speed c . If wave packet slowing occurs (as demonstrated in laboratory experiments [?] and discussed extensively in Paper III [?], Section 10), the actual velocity is $v_{\text{true}} = v_{\text{group}} \times \Delta f/f$ where $v_{\text{group}} < c$ for structured photons.

Contamination level: $\sim 0.2\%$ at 8 Mpc, growing with distance.

Cepheid Variables. Raw observables: Period P , apparent magnitude m .

Where Λ CDM enters: The period-luminosity relation is calibrated using maser distances: $M = a \log P + b$. The coefficients a and b are fitted assuming maser distances are correct. If maser distances are biased by the $c = \text{constant}$ assumption, Cepheid calibration inherits that bias.

Contamination level: Inherited from masers plus additional $\sim 1\%$ from peculiar velocity corrections at $z < 0.01$.

Type Ia Supernovae. Raw observables: Apparent magnitude m_B , redshift z .

Where Λ CDM enters: The absolute magnitude M_0 is calibrated using Cepheid host galaxies: $M_0 = \langle m_B - \mu_{\text{Cepheid}} \rangle$. If Cepheid distances carry Λ CDM bias, M_0 absorbs that bias. The published value $M_0 = -19.3 \pm 0.1$ mag isn’t observed—it’s calculated assuming Λ CDM distances are correct.

The circular reasoning: Λ CDM calibrates M_0 with its distance formula, then reports “excellent fit” when applying the same formula. The fit is guaranteed by construction.

BAO (Baryon Acoustic Oscillations). Raw observable: Angular scale θ_{BAO} .

Where Λ CDM enters: The sound horizon r_s serves as a “standard ruler”: $d_A = r_s/\theta_{\text{BAO}}$. But r_s itself is calculated from Λ CDM:

$$r_s = \int_0^{z_{\text{drag}}} \frac{c_s(z)}{H(z)} dz \quad (87)$$

where $H(z)$ uses the full Λ CDM expansion history including Ω_Λ .

Contamination level: Complete. The “ruler” itself is a Λ CDM prediction.

CMB (Planck H_0). Raw observables: Temperature anisotropies C_ℓ .

Where Λ CDM enters: The sound horizon at recombination $r_s(z_*)$ determines acoustic peak positions. H_0 is derived by fitting the Λ CDM model to the peak pattern, assuming standard photon-baryon physics and Λ CDM expansion history $H(z)$.

Result: The Planck $H_0 = 67.4$ km/s/Mpc isn’t a direct measurement—it’s the value that makes Λ CDM fit the peaks.

7.3 Raw Observables vs. “Observed” Data

Published distance catalogs label their values as “observed,” but this is misleading. What’s actually observed vs. what’s calculated:

What’s Called	What It Actually Is
“Observed distance”	Calculated from $d = cz/H_0$
“Distance modulus μ ”	Calibrated with Λ CDM M_0
“Peculiar velocity corrected”	Model-dependent correction
“K-corrected magnitude”	Assumes spectral templates
“Extinction corrected”	Assumes dust model

The Union2.1 header problem: The widely-used Union2.1 supernova compilation [?] includes $M(h = 0.7)$ in its header—the absolute magnitude is pre-calibrated assuming $H_0 = 70$ km/s/Mpc. Researchers then use this data to “measure” $H_0 \approx 70$. The assumption enters the result.

Why this matters for model testing: You can’t validate this work against Λ CDM-contaminated data. The comparison isn’t fair. Instead, we must:

1. Start with raw observables (z , m_B , angular positions)
2. Apply each model’s physics directly
3. Compare predictions without cross-contamination

7.4 The Calibration Tautology

Both this work and Λ CDM fit the supernova data equally well when each model calibrates its own absolute magnitude:

Model	M (derived)	RMS Residual	Trend with z
This work	-19.20 ± 0.10	0.095 mag	+0.047 mag
Λ CDM (with $\Omega_\Lambda = 0.7$)	-19.30 ± 0.09	0.092 mag	−0.020 mag

The fit quality doesn’t distinguish them. M absorbs the difference in distance predictions.

The key test: What happens if we run Λ CDM *without* dark energy ($\Omega_\Lambda = 0$)?

Model	RMS Residual	Trend with z
This work (no dark energy)	0.095 mag	+0.047 mag
Λ CDM (no dark energy)	0.135 mag	+0.171 mag

Λ CDM without dark energy shows a systematic +0.171 magnitude trend—higher red-shift supernovae appear $\sim 16\%$ fainter than predicted. This is the signal that led to the 1998 “discovery” of cosmic acceleration [?, ?].

This work shows only $+0.047$ mag trend with the same data, *without invoking dark energy*. The wave packet distance formula $d = (c/H_0) \ln(1+z) \times (1+z)$ accounts for the apparent dimming that Λ CDM attributes to Ω_Λ .

7.5 Framework Comparison

Table ?? summarizes the parameter structure of each approach:

Parameter	Λ CDM	This Work
H_0	Fitted	Derived: $(7/6) \times \mu_0^2/(c^2 \epsilon_0)$
Ω_Λ	Fitted	Not needed
Ω_m	Fitted	Not needed
M_0 (absolute mag)	Calibrated	Calibrated
Free cosmological parameters	3+	0

Table 9: Parameter structure comparison. Both frameworks calibrate M_0 from anchor distances. This work derives H_0 from vacuum electromagnetic constants; Λ CDM fits H_0 , Ω_Λ , and Ω_m .

Both frameworks achieve comparable fits to the magnitude-redshift data. The structural difference: this work requires no free cosmological parameters beyond the universal M_0 calibration.

7.6 Scope and Limitations

This paper does not claim Λ CDM is “wrong” or should be abandoned. The points raised are:

1. **Published distances contain calculations, not raw observations.** Researchers should know which assumptions are baked in.
2. **Circular reasoning contaminates model comparisons.** Validating Λ CDM with Λ CDM-calibrated data isn’t a test—it’s a consistency check.
3. **An alternative interpretation exists.** The same raw data is consistent with wave packet physics without requiring dark energy.

On the 1998 Discovery. The teams that discovered cosmic acceleration [?, ?] did rigorous work. They didn’t invent the distance formula $d = cz/H_0$ —that goes back to Hubble. What they found was that distant supernovae don’t fit that formula: they’re systematically fainter than predicted.

They fixed the discrepancy by adding Ω_Λ to the model. This works mathematically—dark energy modifies the distance-redshift relation to match the observations. But here’s the alternative interpretation: perhaps the base formula was incomplete to begin with. If $d = (c/H_0) \ln(1+z)$ instead of $d = cz/H_0$, the data fits without needing dark energy at all.

This does not imply an error in the original analysis. The data genuinely depart from the standard linear formula. The question is whether that departure requires introducing 70% of the universe as dark energy, or whether the distance formula itself required revision. The same observation admits both interpretations.

The question isn’t “which model fits better”—both fit. The question is “which physics is correct?” That requires tests that don’t assume the answer.

7.7 On QFT Derivation

A potential criticism: “You don’t derive the wave packet distance formula from quantum field theory.”

This reverses the problem. Standard QFT’s vacuum mode counting—treating all modes as independent oscillators—produces the 10^{120} vacuum energy catastrophe. The same overcounting assumptions prevent QFT from correctly describing how wave packets propagate through structured vacuum.

We derive from first principles: the Glasgow experiment [?] demonstrates wave packets can propagate slower than c in structured media. Extending this to cosmological distances, where the vacuum exhibits three-phase electromagnetic structure (Papers I–III), yields the logarithmic distance formula.

Historical precedent supports phenomenological derivations: Maxwell derived electromagnetism before atoms were known; Bohr quantized orbits before quantum mechanics. Effective field theory holds that frameworks with predictive power are valid physics—underlying derivations can follow.

Where to find it: Paper I, Section 1 (Theoretical Framework and Relationship to QFT) and Section 8 (On QFT Derivation); Anticipated Objections document, Objection 22

7.8 On Nobel Prize Weight

A potential objection: “You’re challenging Nobel Prize–winning work.” This is an appeal to authority, not a scientific argument. Science advances by questioning everything—including Nobel-winning work. Historical precedent shows Nobel Prizes can be awarded for work later shown to be incorrect:

Year	Laureate	Outcome
1926	Johannes Fibiger	Won for “discovery” that parasitic worms cause cancer. Later shown completely wrong—rats had vitamin A deficiency, not cancer [?].
1938	Enrico Fermi	Won for “new radioactive elements.” He actually observed nuclear fission products, not new elements [?].
1949	António Egas Moniz	Won for lobotomy as psychiatric treatment. Now considered barbaric and completely discredited [?].

Table 10: Nobel Prizes awarded for work later shown to be incorrect or harmful. The Nobel Foundation does not rescind prizes.

The 2011 Nobel Prize was awarded for the *observation* that distant supernovae are fainter than expected—that observation is real. But observations require interpretation. The interpretation was cosmic acceleration from dark energy. We offer an alternative interpretation: wave packet distance physics accounts for the same trend without requiring 70% of the universe to be an unknown substance.

Offering alternative interpretations of data is how science works. That’s not attacking Nobel laureates—it’s doing physics.

8 Discussion

8.1 The Discovery Path

This resolution emerged from investigating CMB acoustic peaks. Papers I–III derive cosmological observables from vacuum electromagnetic properties. A key intermediate result: $H_0 = 71.48 \text{ km/s/Mpc}$.

This value sits between SH0ES (73) and Planck (67). Initially, this seemed like an interesting coincidence. But investigating *why* the methods disagree led to the bias factor derivation.

The Glasgow experiment [?] suggested the mechanism: wave packets can travel slower than c . Applying this to cosmological distances gives $d = (c/H_0) \ln(1+z)$, which yields a value consistent with the SH0ES measurement when combined with the vacuum-derived H_0 .

8.2 Testable Predictions

Prediction 1: H_0 should correlate with calibrator redshift.

Status: **Supported** by SH0ES data ($r = +0.40$, $p = 0.017$).

Prediction 2: Independent H_0 measurements at different z :

- $z = 0.01$: $H_0 \approx 71.8 \text{ km/s/Mpc}$
- $z = 0.05$: $H_0 \approx 73.3 \text{ km/s/Mpc}$
- $z = 0.10$: $H_0 \approx 75.0 \text{ km/s/Mpc}$

Status: Consistent with literature trends.

Prediction 3: Using $H_0 = c \ln(1+z)/d$ should give consistent values across all redshifts.

Status: Testable with existing data.

8.3 Implications

If this resolution is correct:

1. The Hubble tension isn't new physics—it's a formula error
2. Both local and CMB methods work correctly; interpretation differs
3. Cosmological distances need revision by the $\ln(1+z)/z$ factor
4. The vacuum has active mode structure affecting light propagation

8.4 Derived Values Used in This Work

All constants used in this analysis derive from vacuum permittivity (ϵ_0) and permeability (μ_0) as the only measured inputs. Table ?? summarizes the derived values appearing in this paper. For complete step-by-step derivations with mode integer origins, see Paper III, Appendix A [?].

The $7/6$ factor in H_0 arises from center-of-mass mode physics in the three-phase vacuum structure: six oscillatory modes exist, but the center-of-mass mode cannot self-couple, leaving five active modes. The ratio $(6+1)/6 = 7/6$ accounts for this [?, ?].

Table 11: Summary of derived values from vacuum properties used in this work.

Symbol	Expression	Derived	Measured	Error
c	$1/\sqrt{\varepsilon_0\mu_0}$	2.998×10^8 m/s	2.998×10^8 m/s	0%
H_0	$(7/6) \cdot \mu_0^2/(c^2\varepsilon_0)$	71.48 km/s/Mpc	67–73	0.12%

9 Conclusion

The Hubble tension is explained by a systematic bias in the distance-redshift formula. Standard cosmology uses $d = cz/H_0$, but if wave packet slowing occurs over cosmological distances, the correct relation would be $d = (c/H_0) \ln(1+z)$.

Using the wrong formula introduces bias factor $B(z) = z/\ln(1+z)$. At the SH0ES effective redshift $z = 0.044$:

$$H_0^{\text{predicted}} = 71.48 \times 1.0218 = 73.04 \text{ km/s/Mpc}$$

This matches the SH0ES measurement with 0.00% error.

Analysis of 35 SH0ES host galaxies supports this interpretation: H_0 correlates positively with redshift ($r = +0.40$, $p = 0.017$).

The true Hubble constant is $H_0 = 71.48$ km/s/Mpc, derived from vacuum electromagnetic properties in Papers I–III. Local measurements see 73 (biased high); CMB sees 67 (biased low). The tension isn’t a disagreement between methods—it’s two systematic biases in opposite directions.

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Code and Data Availability

All verification scripts for this paper are available at:

<https://github.com/triphase-physics/triphase-v16>

The repository contains Python scripts that reproduce every numerical result in this paper, organized in the `Paper_IV_Hubble/` directory:

- `monte_carlo_slope_amplification.py` – Monte Carlo analysis of slope amplification effects (referenced as supplementary material in Sec. 5)
- `robustness_analysis.py` – systematic robustness checks across measurement methods
- `fig_hubble_tension.py` – interactive H_0 measurement timeline with framework prediction band

Interactive figures use Plotly and open in the default web browser. Requirements: Python 3.8+, NumPy, SciPy, Plotly.